On Nambu Monopole Dynamics in the $SU(2)$ Lattice Higgs Model

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ABSTRACT

It is shown that the $SU(2)$ Higgs model on a lattice is equivalent to the Georgi–Glashow model in the limit of a small coupling constant between the Higgs and gauge fields. It can therefore be concluded that the transition between the confinement and symmetric phases in the $3 + 1$ dimensional $SU(2)$ Higgs model at finite temperature is accompanied by condensation of Nambu monopoles.

According to one of the most popular modern hypotheses, the baryon asymmetry of the universe may arise in the process of an electroweak phase transition (see, for example, the review by Rubakov and Shaposhnikov [1]).

Due to the smallness of the Weinberg angle θ_W and the insignificance of the fermion effects, this transition is largely determined by the properties of the $SU(2)$ Higgs model. The present letter examines the behavior of the magnetic fluctuations which can play an important role in a temperature phase transition in the SU(2) Higgs model.

Let us consider the SU(2) lattice Higgs model with scalar field Φ_x in the fundamental representation, the action is:

$$S[U, \Phi] = -\frac{\beta}{2} \sum_P \mathrm{Tr} U_P - \frac{\kappa}{2} \sum_x \sum_\mu \left(\Phi_x^+ U_{x,\mu} \Phi_{x+\hat{\mu}} + c.c. \right) + V(|\Phi|), \qquad (1)$$

Here U_P represents the ordered product of the edge elements of the gauge field $U_{x,\mu}$ over the boundaries faces of the plaquette P, and $V(\Phi)$ is the potential on the field Φ, and $|\Phi|^2 = \Phi^+\Phi$.

Due to the triviality of the homotopy group $\pi_2(SU(2))$, there are no topologically stable monopole defects in this theory. However, "embedded" [2] monopoles, the so-called "Nambu monopoles" [3], which are not topologically stable defects, do exist in

the theory. These objects are described by the composite field

$$\chi_x^a = \Phi_x^+ \sigma^a \Phi_x\,, \tag{2}$$

(σ^a are the Pauli matrices) which behaves under gauge transformations as a scalar field in the adjoint representation. A Nambu monopole is a configuration of fields U and Φ such that the field U and the composite field χ, expressed in terms of the fundamental field Φ, according to (2), coincide with the configuration of the 't Hooft–Polyakov monopole [4] in the Georgi–Glashow model [5] with the field χ in the adjoint representation and with the gauge field U. Since Nambu monopoles are described solely by the gauge field U and the composite field χ, the dynamics of these monopoles is determined completely by the effective action S_{eff}:

$$e^{-S_{eff}[U,\chi]} = \int \mathcal{D}\Phi \, e^{-S[U,\Phi]} \prod_a \prod_x \delta(\chi_x^a - \Phi_x^+ \sigma^a \Phi_x)\,. \tag{3}$$

To calculate the action S_{eff} it is convenient to study the following parametrization of the field Φ:

$$\Phi = e^{i\varphi}\,\Psi\,, \qquad \Psi = \rho \begin{pmatrix} \cos\alpha\,e^{i\theta} \\ \sin\alpha \end{pmatrix}\,,$$

where $\varphi, \theta \in [-\pi, \pi)$, $\alpha \in [0, \pi/2]$ and $\rho \in [0, +\infty)$. The fields ρ, α and θ can be expressed in terms of the field χ with the aid of Eq.(2)!:

$$\theta = \arctan\frac{\chi^2}{\chi^1}\,, \qquad \alpha = \frac{1}{2}\arctan\frac{\sqrt{(\chi^1)^2 + (\chi^2)^2}}{|\chi^3|}\,, \qquad \rho = \sqrt{|\chi|}\,,$$

whence

$$\Psi = \frac{1}{\sqrt{2\left(|\chi| - |\chi^3|\right)}} \cdot \begin{pmatrix} \chi^1 + i\chi^2 \\ |\chi| - |\chi^3| \end{pmatrix}\,. \tag{4}$$

Using the relation for the modulus of the field Φ, $|\Phi|^2 = |\chi| = \sqrt{(\sum_{a=1}^{3} (\chi^a)^2)}$, and the measure

$$\int\limits_{-\infty}^{+\infty} \mathcal{D}\Phi \cdots = \int\limits_{-\pi}^{\pi} \mathcal{D}\varphi \cdot \int\limits_{-\infty}^{+\infty} \prod_x \frac{1}{|\chi_x|} \mathrm{d}\chi_x^1 \mathrm{d}\chi_x^2 \mathrm{d}\chi_x^3 \cdots\,,$$

we get for the effective action (3):

$$S_{eff}[U, \chi] = -\frac{\beta}{2} \sum_P \mathrm{Tr}U_P + S_h[U, \chi] + \tilde{V}(|\chi|)\,, \tag{5}$$

where the new potential on the field χ is determined by the expression

$$\tilde{V}(|\chi|) = V(\sqrt{|\chi|}) + \sum_x \ln|\chi_x|\,, \tag{6}$$

and the interaction of the fields U and χ is

$$e^{-S_h[U,\chi]} = \int\limits_{-\pi}^{\pi} \mathcal{D}\varphi \exp\Big\{\kappa \sum_x \sum_\mu R_{x,\mu} \cos(\varphi_{x+\hat{\mu}} - \varphi_x + A_{x,\mu})\Big\}\,. \tag{7}$$

In this formula we introduced the notation

$$\Psi_x^+ U_{x,\mu} \Psi_{x+\hat{\mu}} = R_{x,\mu} e^{iA_{x,\mu}}\,. \tag{8}$$

The derivation of the effective action (5) is correct in any dimension of space–time.

For simplicity, we shall examine the case of an infinitely deep potential $V(|\Phi|)$ with the minimum $|\Phi|^2 = |\chi| = 1$. In this case, the lengths of the Higgs field Φ and of the composite field χ are frozen. The integral (7) is most easily calculated in the limit $\kappa \ll 1$. In leading order we obtain (to within an additive constant):

$$S_h = -\frac{\kappa^2}{2} \sum_x \sum_\mu R_{x,\mu}^2 + O(\kappa^4) = -\frac{\kappa^2}{8} \sum_x \sum_\mu \mathrm{Tr}(U_{x,\mu}\,\chi_x\,U_{x,\mu}^+\,\chi_{x+\hat{\mu}}) + O(\kappa^4)\,,$$

where we employed Eqs.(4) and (8) and introduced the notation $\chi = \chi^a \sigma^a$. Thus in the limit $\kappa \ll 1$ the effective action (5) with the length $|\Phi|^2 = 1$ of the Higgs field frozen is identical in leading order to the Georgi–Glashow action:

$$S_{eff}[U,\chi] = -\frac{\beta}{2} \sum_P \mathrm{Tr}U_P - \frac{\gamma}{2} \sum_x \sum_\mu \mathrm{Tr}(U_{x,\mu}\,\chi_x\,U_{x,\mu}^+\,\chi_{x+\hat{\mu}}) + O(\kappa^4)\,, \tag{9}$$

where

$$\gamma = \frac{\kappa^2}{4}\,. \tag{10}$$

It is interesting to compare the phase diagrams of the $3+1$–dimensional $SU(2)$ Higgs model (1) and the Georgi–Glashow model (9,10) at nonzero temperature with radially frozen Higgs fields. Figure 1 displays schematically the phase diagram obtained in Ref. [7] for the $SU(2)$ Higgs model. Figure 2 shows the phase diagram obtained in Ref. [8] for the Georgi–Glashow model. For small values of the constant β both theories are in the confinement phase (color confinement). As β increases, a phase transition from the confinement phase to the symmetric phase occurs in both theories at small κ. The line of phase transitions A'-B' in the Georgi–Glashow model should correspond to the line of phase transitions A-B in the $SU(2)$ Higgs model according to Eq. (10): $\gamma_c(\beta) = \kappa_c^2(\beta)/4 + O(\kappa_c^4)$.

Figure 2 shows schematically the phase transition predicted with the aid of Eq. (10) (dashed line A'-C'). Unfortunately, it is impossible to determine the correctness of this prediction quantitatively on the basis of the results of Refs. [7] and [8], since in those papers the phase diagrams were studied at different temperatures.

It is known [6] that in the Georgi–Glashow model confinement is due to the dynamics of the 't Hooft–Polyakov monopoles: in the confinement phase the monopoles are condensed, while in the deconfinement phase there exists a dilute gas of monopole–anti-monopole pairs. Therefore, at least for low values of the coupling constant κ, the phase transition from the symmetric phase to the confinement phase in the $SU(2)$ Higgs model (1) is accompanied by condensation of Nambu monopoles, since the Nambu monopoles in the $SU(2)$ Higgs model (1) correspond to the 't Hooft–Polyakov monopoles in the Georgi–Glashow model (9).

It is natural to suppose that condensation of Nambu monopoles also occurs for larger values of the parameter κ for the phase transitions from the confinement phase to the symmetric phase and from the confinement phase to the Higgs phase. The latter conjecture finds support in the fact that in the Higgs phase there exists an embedded string, the so-called Z-vortex string [3, 9], with nonzero string tension. Stretched between Nambu monopoles, such a string destroys the condensate. The results of investigations of this question will be published later in a separate paper.

Note Added

The dynamical role of these embedded topological defects has been further studied in Refs. [10, 11, 12] in a $3D$ $SU(2)$ model which is an effective dimensionally reduced version of the Electroweak theory [13, 14, 15]. The Z–vortices exhibit a percolation transition which accompanies the first order thermal transition at small Higgs mass [10]. The percolation transition exists also in the cross-over regime at high Higgs masses [11]. Ref. [12] gives a short review of these and other related results.

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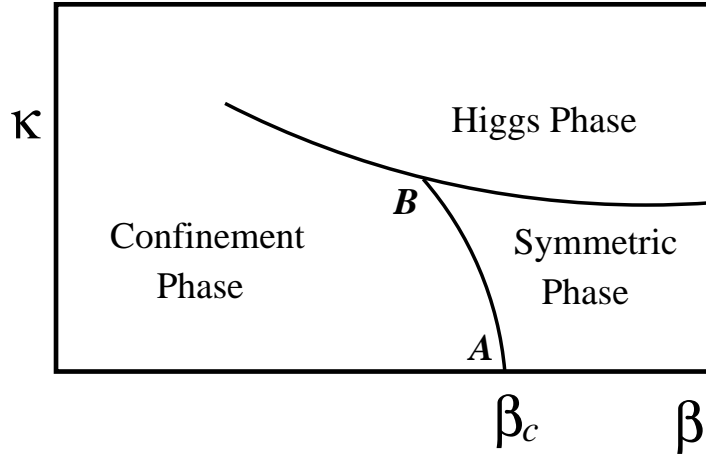1998; `hep-lat/9809045`.

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Figures

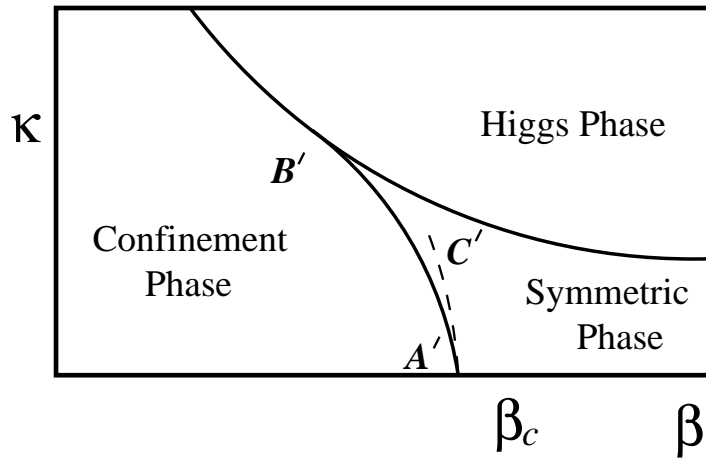Figure 1: Phase diagram of the $SU(2)$ Higgs model at nonzero temperature, the Higgs field radius is fixed.

Figure 2: The same as in Figure 1 but for the Georgi–Glashow model.